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Ecole Supérieure Polytechnique
**Institut Préparatoire aux Grandes Ecoles d'Ingénieurs**

الجمهوريةالاسلاميةالموريتانية
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المد رسة العليا للمتعدد التقنيات
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**Nom :** ------------------------------------------------------------------------

**Prénom :** ------------------------------------------------------------------------

| Signature du candidat | Numéro d'inscription | Matricule |
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**N° Anonymat**

**N° Anonymat**

# CONCOURS NATIONAL D'INGÉNIEURS DE MAURITANIE

(CNIM)

# CONCOURS 2017

## ÉPREUVE DES SCIENCES DE L'INGENIEUR

Durée de l'épreuve : 3 heures

**L'usage de l'ordinateur, de la calculatrice ou tout autre objet connecté est interdit.**

*Le sujet comporte:*

- *11 pages d'énoncé*
- *12 pages de document réponse*

*Si au cours de l'épreuve, le candidat repère ce qui lui semble être une erreur d'énoncé, il le signale sur sa copie et poursuit sa composition en expliquant les raisons des initiatives qu'il est amené à prendre.*

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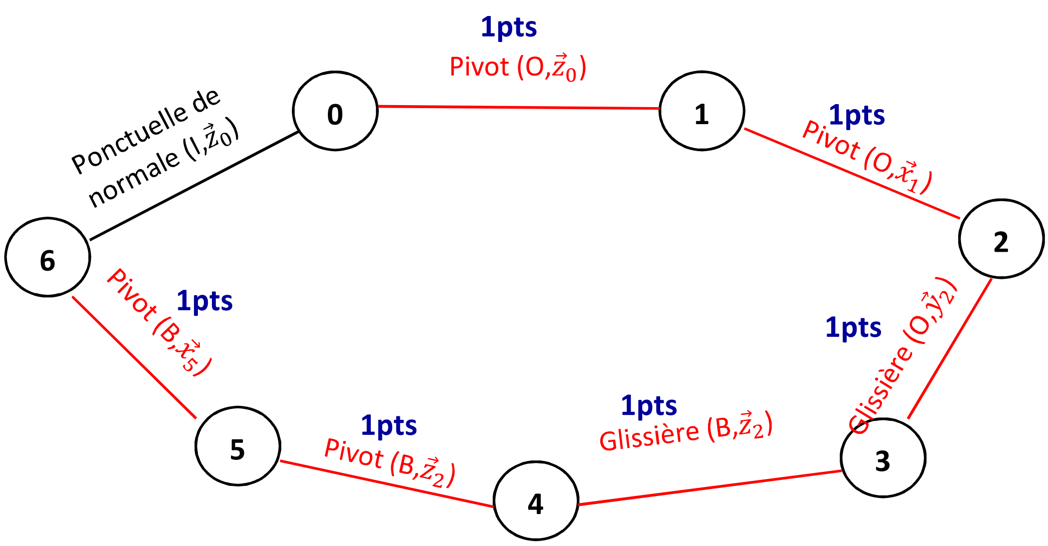
## Partie 1. Validation des capacités d'adaptation de la passerelle à son environnement

**Question 1.** **8pts**




**2pts pour la chaine**

| Question 2. | 8pts |
|---|---|

$\vec{y}_2 = \cos\beta\,\vec{y}_1 + \sin\beta\,\vec{z}_0 = \cos\beta\,(-\sin\alpha\,\vec{x}_0 + \cos\alpha\,\vec{y}_0) + \sin\beta\,\vec{z}_0$

$\vec{y}_2 = -\sin\alpha\cos\beta\,\vec{x}_0 + \cos\alpha\cos\beta\,\vec{y}_0 + \sin\beta\,\vec{z}_0$

$\vec{z}_2 = -\sin\beta\,\vec{y}_1 + \cos\beta\,\vec{z}_0 = -\sin\beta\,(-\sin\alpha\,\vec{x}_0 + \cos\alpha\,\vec{y}_0) + \cos\beta\,\vec{z}_0$

$\vec{z}_2 = \sin\alpha\sin\beta\sin\alpha\,\vec{x}_0 - \cos\alpha\sin\beta\,\vec{y}_0 + \cos\beta\,\vec{z}_0$

4pts

$$\vec{y}_2 = \begin{bmatrix} \ldots -\sin\alpha\cos\beta \ldots \\ \ldots \cos\beta\cos\alpha \ldots \\ \ldots \sin\beta \ldots \end{bmatrix}_{R_0}$$

4pts

$$\vec{z}_2 = \begin{bmatrix} \ldots \sin\beta\sin\alpha \ldots \\ \ldots -\sin\beta\cos\alpha \ldots \\ \ldots \cos\beta \ldots \end{bmatrix}_{R_0}$$

| Question 3. | 8pts |
|---|---|

$\overrightarrow{OI} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BI}$

2pts

$x_I\vec{x}_0 + y_I\vec{y}_0 + z_I\vec{z}_0 = \lambda(t)\vec{y}_2 - \mu(t)\vec{z}_2 - \frac{D}{2}\vec{z}_0$

$$\begin{pmatrix} x_I \\ y_I \\ z_I \end{pmatrix}_{R_0} = \lambda(t)\begin{pmatrix} -\sin\alpha\cos\beta \\ \cos\beta\cos\alpha \\ \sin\beta \end{pmatrix}_{R_0} - \mu(t)\begin{pmatrix} \sin\beta\sin\alpha \\ -\sin\beta\cos\alpha \\ \cos\beta \end{pmatrix}_{R_0} - \frac{D}{2}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{R_0}$$

$$\begin{cases} x_I = -\lambda(t)\sin\alpha\cos\beta - \mu(t)\sin\beta\sin\alpha \\ y_I = \lambda(t)\cos\beta\cos\alpha + \mu(t)\sin\beta\cos\alpha \\ z_I = \lambda(t)\sin\beta - \mu(t)\cos\beta - \frac{D}{2} \end{cases}$$

$$\begin{cases} x_I = \cdots \ldots - \lambda(t)\sin\alpha\cos\beta - \mu(t)\sin\beta\sin\alpha \ldots\ldots\ldots\ldots (1) \\ y_I = \cdots \ldots\ldots \lambda(t)\cos\beta\cos\alpha + \mu(t)\sin\beta\cos\alpha \ldots\ldots\ldots (2) \\ z_I = \cdots \ldots\ldots\ldots \lambda(t)\sin\beta - \mu(t)\cos\beta - \dfrac{D}{2} \ldots\ldots\ldots\ldots\ldots (3) \end{cases}$$

2pts

2pts

2pts

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| Question 4. | 8pts |
|---|---|

$\overrightarrow{ID} = \overrightarrow{IB} + \overrightarrow{BA} + \overrightarrow{AD}$ **2pts**

$x_D\vec{x}_0 + y_D\vec{y}_0 + z_D\vec{z}_0 = \frac{D}{2}\vec{z}_0 + \mu(t)\vec{z}_2 + l\vec{y}_2$

$$\begin{pmatrix} x_D \\ y_D \\ z_D \end{pmatrix}_{R_0} = \frac{D}{2}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{R_0} + \mu(t)\begin{pmatrix} \sin\beta\sin\alpha \\ -\sin\beta\cos\alpha \\ \cos\beta \end{pmatrix}_{R_0} + l\begin{pmatrix} -\sin\alpha\cos\beta \\ \cos\beta\cos\alpha \\ \sin\beta \end{pmatrix}_{R_0}$$

$$\begin{cases} x_D = \mu(t)\sin\beta\sin\alpha - l\sin\alpha\cos\beta \\ y_D = -\mu(t)\sin\beta\cos\alpha + l\cos\beta\cos\alpha \\ z_D = \frac{D}{2} + \mu(t)\cos\beta + l\sin\beta \end{cases}$$

$$\begin{cases} x_D = \cdots \ldots\ldots\ldots \mu(t)\sin\beta\sin\alpha - l\sin\alpha\cos\beta \ldots\ldots\ldots.. (4) \\ y_D = \cdots \ldots\ldots\ldots - \mu(t)\sin\beta\cos\alpha + l\cos\beta\cos\alpha \ldots\ldots.. (5) \\ z_D = \cdots \ldots\ldots\ldots\ldots \frac{D}{2} + \mu(t)\cos\beta + l\sin\beta \ldots\ldots\ldots\ldots\ldots.. (6) \end{cases}$$

**2pts**
**2pts**
**2pts**

| Question 5. | 8pts |
|---|---|

$\alpha = 0 \rightarrow x_D = 0$ et $x_I = 0$ **1pts**

$$\begin{cases} y_I = \lambda(t)\cos\beta + \mu(t)\sin\beta = y_F \\ z_I = \lambda(t)\sin\beta - \mu(t)\cos\beta - \frac{D}{2} = -h \\ y_D = -\mu(t)\sin\beta + l\cos\beta = y_a \\ z_D = \frac{D}{2} + \mu(t)\cos\beta + l\sin\beta = z_a \end{cases}$$

**1pts**
**1pts**
**1pts**
**1pts**

A partir de ce système, on a :

$$\begin{cases} z_a - h = (l + \lambda(t))\sin\beta \\ y_a + y_F = (l + \lambda(t))\cos\beta \end{cases}$$

**1pts**
**1pts**

D'où :

$\tan\beta = \frac{z_a - h}{y_a + y_F}$ **1pts**

| Question 6. | 8pts |
|---|---|

$\tan\beta = \frac{z_a - h}{y_a + y_F} = \frac{2-4}{38+2} = -5\%$ **4pts**

Le cahier des charges est respecté **4pts**

## Partie 2. Validation de la charge maximale admissible par l'essieu de la passerelle

| Question 7. | 8pts |
|---|---|

$\overrightarrow{OG} = \frac{1}{\sum_i m_i} \sum_i m_i \overrightarrow{OG_i}$ **4pts**

$y_G = \frac{m_2 y_2 + m_3 y_3}{m_2 + m_3}$ **4pts**

| Question 8. | 8pts |
|---|---|

$\bar{S} = \{0, 4, g\}$

$\{F(0 \to S)\} = {}_{O}\begin{Bmatrix} X & 0 \\ Y & M \\ Z & N \end{Bmatrix}_{R_0}$ **2pts**

$\overrightarrow{M_O}(4 \to S) = \overrightarrow{M_B}(4 \to S) + \vec{\mathcal{R}}(4 \to S) \wedge \overrightarrow{BO} = F\vec{z}_2 \wedge (a\vec{z}_2 - b\vec{y}_2) = bF\vec{x}_0$

$\{F(4 \to S)\} = {}_{O}\begin{Bmatrix} 0 & bF \\ F\sin\beta & 0 \\ F\cos\beta & 0 \end{Bmatrix}_{R_0}$ **2pts**

$\overrightarrow{M_O}(g \to S) = \overrightarrow{OG} \wedge M\vec{g} = y_G\vec{y}_2 \wedge (-Mg\vec{z}_0) = -Mgy_G \cos\beta\, \vec{x}_0$

$\{F(4 \to S)\} = {}_{O}\begin{Bmatrix} 0 & -Mgy_G\cos\beta \\ 0 & 0 \\ -Mg & 0 \end{Bmatrix}_{R_0}$ **2pts**

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$$\{F(\bar{S} \to S)\} = \left\{ \begin{matrix} \ldots\ldots X \ldots\ldots\ldots \\ \ldots\ldots Y + F\sin\beta \ldots\ldots\ldots \\ \ldots\ldots Z + F\cos\beta - Mg \ldots\ldots \end{matrix} \right|_O \left. \begin{matrix} \ldots bF - Mgy_G\cos\beta \ldots \\ \ldots\ldots M \ldots\ldots \\ \ldots\ldots N \ldots\ldots\ldots \end{matrix} \right\}_{R_0}$$ **2pts**

| **Question 9.** | **8pts** |
|---|---|

$\vec{x}_0.\overrightarrow{M_0}(\bar{S} \to S) = 0$ **4pts**

$bF - Mgy_G\cos\beta$

$F = \frac{Mgy_G\cos\beta}{b}$ **4pts**

| **Question 10.** | **8pts** |
|---|---|

$F = \frac{Mgy_G\cos\beta}{b} = \frac{60.10^3 \times 10 \times 10}{15} = 4.10^5 N = 40 tonnes$ **4pts**

$F = 40 tonnes < 70 tonnes$ **2pts**

Le cahier des charges est respecté **2pts**

## Partie 3. Validation de l'aptitude de la passerelle au déplacement horizontal

| **Question 11.** | **8pts** |
|---|---|

$\vec{V}(I \in 6/sol) = \vec{V}(B \in 6/sol) + \vec{\Omega}(6/sol) \wedge \overrightarrow{BI} = \dot{y}(t)\vec{y}_0 + \left(\frac{-\omega_m(t)}{K_r}\vec{x}_0\right) \wedge \left(\frac{-D}{2}\vec{z}_0\right)$

$\vec{V}(I \in 6/sol) = \left(\dot{y}(t) - \frac{D\omega_m(t)}{2K_r}\right)\vec{y}_0$ **2pts**

Condition de roulement sans glissement : $\vec{V}(I \in 6/sol) = \vec{0}$

$\dot{y}(t) = \frac{D\omega_m(t)}{2K_r}$ **2pts**

$\omega_{max} = \frac{2K_r}{D}\dot{y}_{max}$ **2pts**

AN : $\omega_{max} = \frac{2 \times 2000}{1} \times 1{,}2 = 4800 rad/s = 12{,}73 tr/min$ **2pts**

| Question 12. | 8pts |
|---|---|

$\Sigma = \{3,6\}$

$E_c(3/0) = \frac{1}{2}m_3\dot{y}^2 = \frac{1}{2}m_3\frac{D^2}{4K_r{}^2}\omega_m{}^2(t)$ **2pts**

$E_c(6/0) = \frac{1}{2}m_6\dot{y}^2 + \frac{1}{2}A_6\dot{\theta}^2 = \frac{1}{2}m_6\frac{D^2}{4K_r{}^2}\omega_m{}^2(t) + \frac{1}{2}A_6\frac{\omega_m{}^2(t)}{K_r{}^2} = \frac{1}{2}\frac{\omega_m{}^2(t)}{K_r{}^2}\left(m_6\frac{D^2}{4} + A_6\right)$ **2pts**

$E_c(\Sigma/0) = \frac{1}{2}\frac{\omega_m{}^2(t)}{K_r{}^2}\left((m_3+m_6)\frac{D^2}{4} + A_6\right) = \frac{1}{2}J_{eq}\omega_m{}^2(t)$ avec $J_{eq} = \frac{1}{K_r{}^2}\left((m_3+m_6)\frac{D^2}{4} + A_6\right)$

| |
|---|
| $E_c(\Sigma/R_0) = \frac{1}{2}J_{eq}\omega_m{}^2(t)$ **2pts** |
| $J_{eq}(\Sigma) = \frac{1}{K_r{}^2}\left((m_3+m_6)\frac{D^2}{4} + A_6\right)$ **2pts** |

| Question 13. | 8pts |
|---|---|

$P(\text{int à } \Sigma) = P(6 \leftrightarrow 3) = 0$ car liaison parfaite.

**8pts**

| |
|---|
| $P(\text{int à } \Sigma) = 0$ |

| Question 14. | 8pts |
|---|---|

$\Sigma = \{3,6\}$ ➔ $\bar{\Sigma} = \{0, sol, g, MR_1\}$ **1pts**

$P(\bar{\Sigma} \to \Sigma/R_0) = P(0 \to 3/R_0) + P(\text{sol} \to 6/R_0) + P(\text{g} \to 3/R_0) + P(\text{g} \to 6/R_0) + P(MR_1 \to 6/R_0)$ **1pts**

$P(\text{sol} \to 6/R_0) = 0$ car condition de roulement sans glissement **1pts**

$P(\text{g} \to 3/R_0) = P(\text{g} \to 6/R_0) = 0$ **2pts**

$P(MR_1 \to 6/R_0) = \eta c_m(t)\omega_m(t)$

$P(0 \to 3/R_0) = -f_R(t)\dot{y}(t) = -\frac{D}{2K_r}f_R(t)\omega_m(t)$ **1pts**

$P(\bar{\Sigma} \to \Sigma/R_0) = \left(\eta c_m(t) - \frac{D}{2K_r}f_R(t)\right)\omega_m(t)$ **1pts**

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$$P(\bar{\Sigma} \rightarrow \Sigma/R_0) = \left(\eta c_m(t) - \frac{D}{2K_r} f_R(t)\right) \omega_m(t)$$ **1pts**

| Question 15. | **8pts** |
|---|---|

$$\frac{dE_c(\Sigma/R_0)}{dt} = P(\bar{\Sigma} \rightarrow \Sigma/R_0) + P(\text{int à } \Sigma)$$ **4pts**

$$J_{eq}\omega_m(t)\frac{d\omega_m(t)}{dt} = \omega_m(t)\left(\eta c_m(t) - \frac{D}{2K_r} f_R(t)\right)$$

$$c_m(t) = \frac{1}{\eta}\left(J_{eq}\frac{d\omega_m(t)}{dt} + \frac{D}{2K_r} f_R(t)\right)$$

$$c_m(t) = \frac{1}{\eta}\left(J_{eq}\frac{d\omega_m(t)}{dt} + \frac{D}{2K_r} f_R(t)\right)$$ **4pts**

| Question 16. | **8pts** |
|---|---|

| Période | $[0, T_a]$ | $]T_a, T - T_a[$ | $[T - T_a, T]$ |
|---|---|---|---|
| $\frac{d\omega_m(t)}{dt}$ en $(rads^{-2})$ | $\frac{\omega_{max}}{T_a}$ **1pts** | **1pts** $0$ | **1pts** $-\frac{\omega_{max}}{T_a}$ |
| Couple moteur $c_m(Nm)$ Expression analytique | **1pts** $\frac{1}{\eta}\left(J_{eq}\frac{\omega_{max}}{T_a} + \frac{D}{2K_r} f_R(t)\right)$ | **1pts** $\frac{D}{2\eta K_r} f_R(t)$ | **1pts** $\frac{1}{\eta}\left(-J_{eq}\frac{\omega_{max}}{T_a} + \frac{D}{2K_r} f_R(t)\right)$ |

Le moteur fournit un couple maximal dans la période $[0, T_a]$ **2pts**

## Partie 4. Validation des performances de déplacement horizontal de la passerelle

**Question 17.** **8pts**

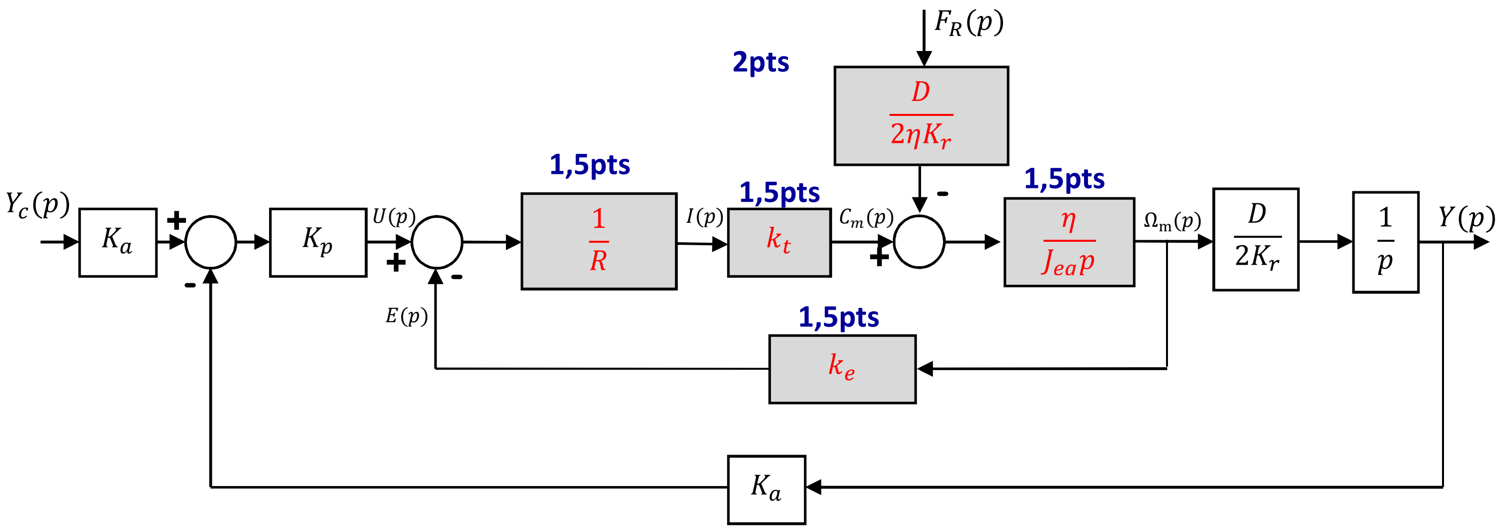
$U(p) = RI(p) + E(p)$ **2pts**

$E(p) = k_e \Omega_m(p)$ **2pts**

$C_m(p) = k_t I(p)$ **2pts**

$C_m(p) = \frac{D}{2\eta K_r} F_R(p) = \frac{J_{eq}}{\eta} p\Omega_m(p)$ **2pts**

**Question 18.** **8pts**




**Question 19.** **8pts**

$H_1(p) = \frac{Y(p)}{Y_c(p)} = \frac{1}{1+\frac{2K_r k_e}{k_p k_a D}p+\frac{2K_r R J_{eq}}{\eta k_p k_a k_t D}p^2}$ **8pts**

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$$H_1(p) = \frac{Y(p)}{Y_c(p)} = \frac{1}{1+\frac{2K_r k_e}{k_p k_a D}p+\frac{2K_r R J_{eq}}{\eta k_p k_a k_t D}p^2}$$

**Question 20.** **8pts**

$$H_1(p) = \frac{1}{1+\frac{2K_r k_e}{k_p k_a D}p+\frac{2K_r R J_{eq}}{\eta k_p k_a k_t D}p^2} = \frac{k_s}{1+\frac{2m}{\omega_0}p+\frac{p^2}{\omega_0{}^2}}$$

$k_s = 1$

$\frac{1}{\omega_0{}^2} = \frac{2K_r R J_{eq}}{\eta k_p k_a k_t D}$ ➔ $\omega_0 = \sqrt{\frac{\eta k_p k_a k_t D}{2K_r R J_{eq}}}$

$\frac{2m}{\omega_0} = \frac{2K_r k_e}{k_p k_a D}$ ➔ $m = k_e\sqrt{\frac{\eta k_r k_t}{2k_p k_a D R J_{eq}}}$

**2pts** **3pts** **3pts**

| | | |
|---|---|---|
| $k_s = 1$ | $\omega_0 = \sqrt{\frac{\eta k_p k_a k_t D}{2K_r R J_{eq}}}$ | $m = k_e\sqrt{\frac{\eta k_r k_t}{2k_p k_a D R J_{eq}}}$ |

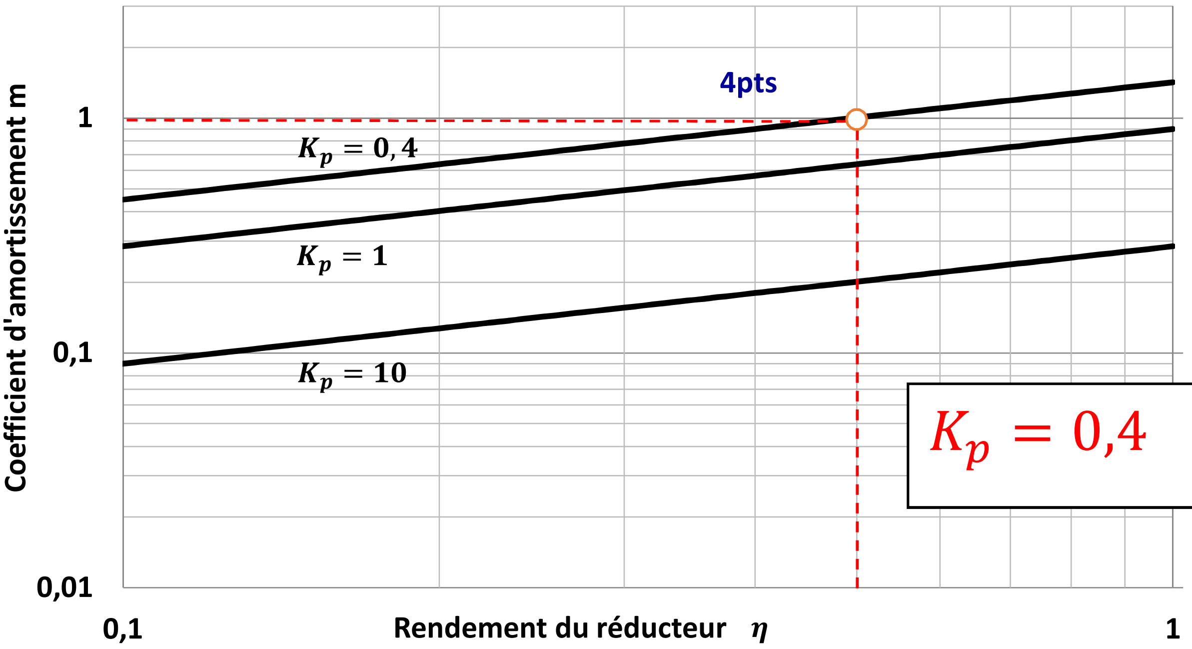
**Question 21.** **8pts**

Réponse rapide sans dépassement ➔ $m = 1$ **4pts**

$m = k_e\sqrt{\frac{\eta k_r k_t}{2k_p k_a D R J_{eq}}} = 1$ ➔ $k_p = \frac{\eta k_r k_t k_e{}^2}{2k_a D R J_{eq}}$ **4pts**

$$k_p = \frac{\eta k_r k_t k_e{}^2}{2k_a D R J_{eq}}$$

**Question 22.** **8pts**




$K_p = 0,4$ 4pts

| Question 23. | 8pts |
|---|---|

$FTBF(p) = \frac{0,0625k_p}{0,0625k_p+p+10p^2}$ 4pts

$$FTBF(p) = \frac{0,0625k_p}{0,0625k_p+p+10p^2}$$

$D(p) = 0,0625k_p + p + 10p^2$

Le système est stable ssi $k_p > 0$ 4pts

| Question 24. | 8pts |
|---|---|

| | |
|---|---|
| $t_{r5\%} = 94s$ 1pts | $\varepsilon_p = 0$ 1pts |

Conclusion :

Critère de précision respecté 2pts

Critère de rapidité non respecté 2pts

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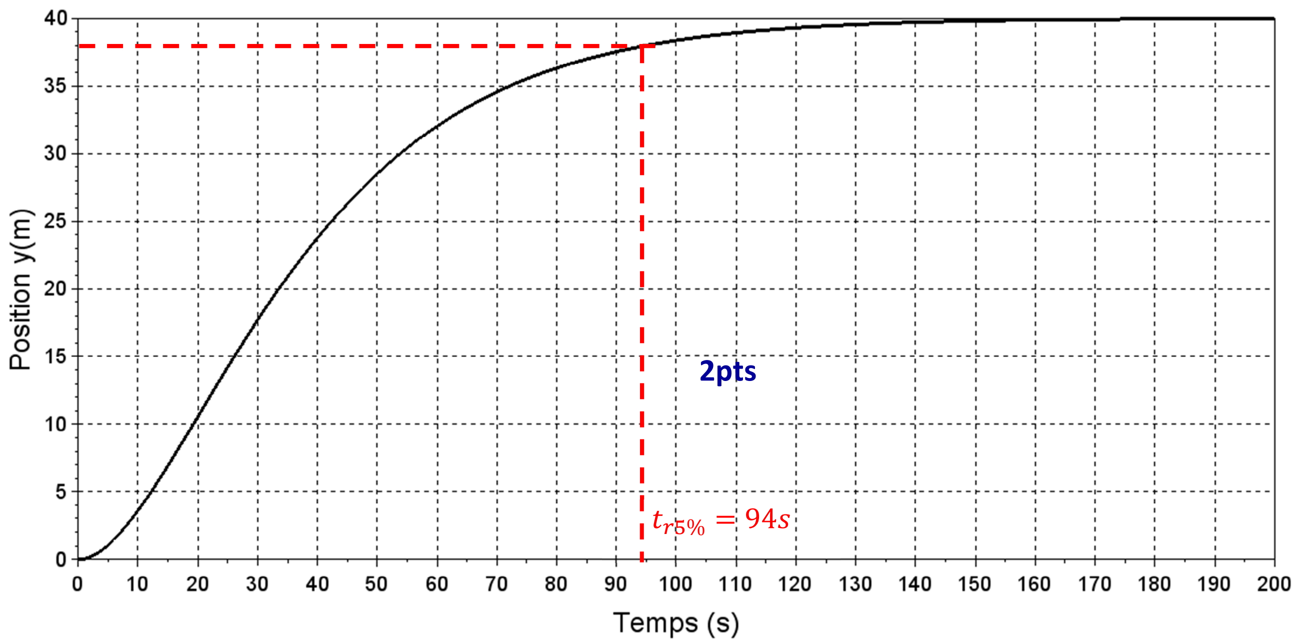
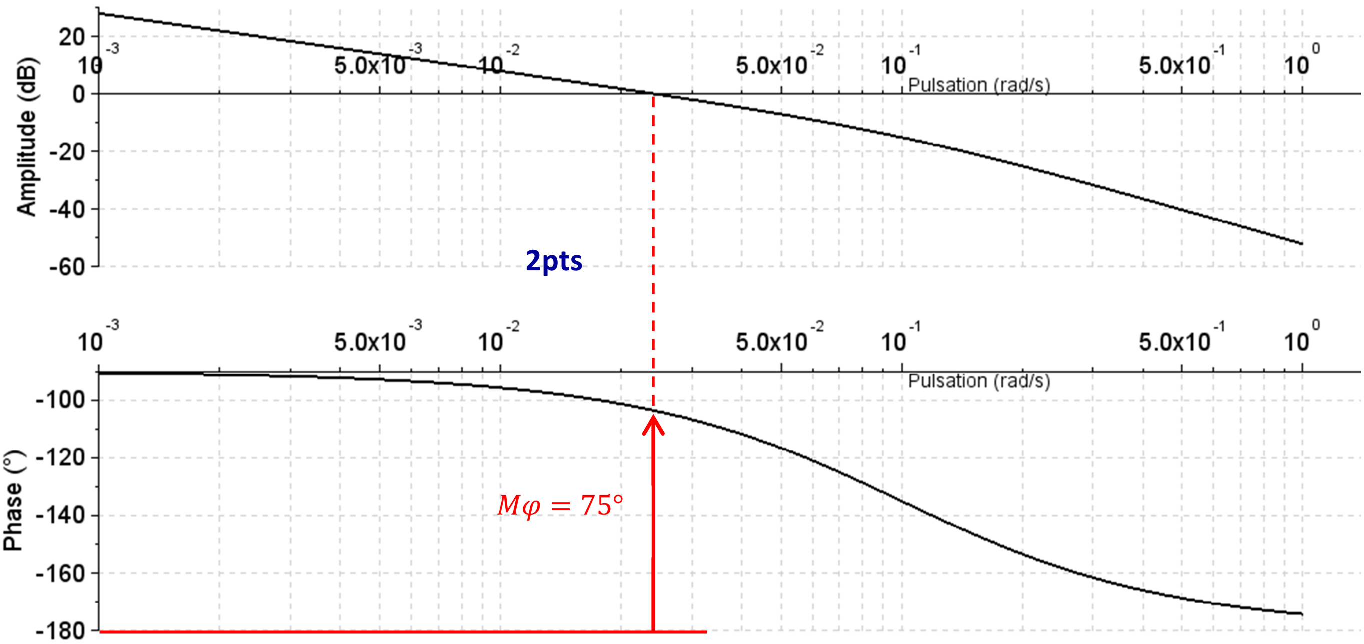



| **Question 25.** | **8pts** |
| --- | --- |

Marge de phase = 75°>45° **2pts**

Marge de gain =+infini **2pts**

Critère de stabilité vérifié **2pts**

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